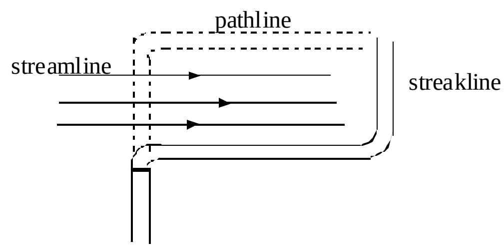
CHAPTER 3

Introduction to Fluids in Motion

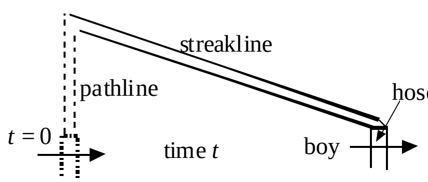
3.1



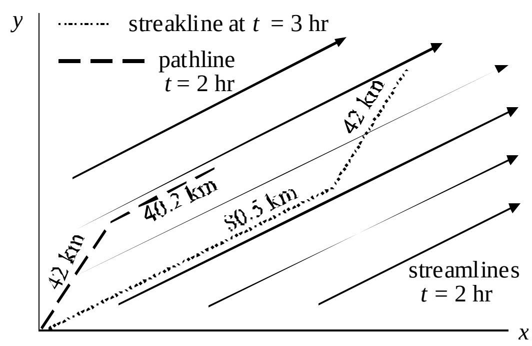
3.2 Pathlines: Release several at an instant in time and take a time exposure of the subsequent motions of the bulbs.

Sreakline: Continue to release the devises at a given location and after the last one is released, take a snapshot of the "line" of bulbs. Repeat this for several different release locations for additional streaklines.

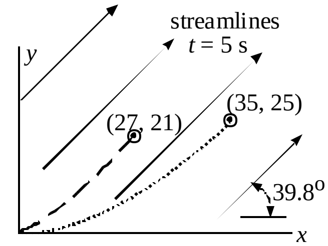
3.3



3.4



3.5 a) $u = \dfrac{dx}{dt} = 2t + 2 \qquad v = \dfrac{dy}{dt} = 2t$



$x = t^2 + 2t + c_1 \qquad y = t^2 + c_2$

$= y + 2\sqrt{y}$

$\therefore x^2 - 2xy + y^2 = 4y \qquad \therefore$ parabola.

b) $x = t^2 + 2t + c_1$. $\therefore c_1 = -8$, and $c_2 = -4$.

$= y + 4 + 2(\pm\sqrt{y+4}) - 8$

$\therefore x^2 - 2xy + y^2 + 8x - 12y = 0. \qquad \therefore$ parabola.

3.6

$$\left.\begin{aligned} \vec{V} &= u\hat{i} + v\hat{j} + w\hat{k} \\ d\vec{r} &= dx\hat{i} + dy\hat{j} + dz\hat{k} \end{aligned}\right\} \qquad (\vec{V} \times d\vec{r})_z = \underline{udy - vdx}$$

using $\hat{i} \times \hat{j} = \hat{k},\ \hat{j} \times \hat{i} = -\hat{k}$.

3.7 Lagrangian: Several college students would be hired to ride bikes around the various roads, making notes of quantities of interest.

Eulerian: Several college students would be positioned at each intersection and quantities would be recorded as a function of time.

3.8 a) At $t = 2$ and (0,0,0) $\quad V = \sqrt{2^2} = \underline{2\text{ m/s}}$.

At $t = 2$ and (1,–2,0) $\quad V = \sqrt{3^2 + 2^2} = \underline{3.606\text{ m/s}}$.

b) At $t = 2$ and (0,0,0) $\quad V = \underline{0}$.

At $t = 2$ and (1,–2,0) $\quad V = \sqrt{(-2)^2 + (-8)^2} = \underline{8.246\text{ m/s}}$.

c) At $t = 2$ and (0,0,0) $\quad V = \sqrt{(-4)^2} = \underline{4\text{ m/s}}$.

At $t = 2$ and (1,–2,0) $\quad V = \sqrt{2^2 + (-4)^2 + (-4)^2} = \underline{6\text{ m/s}}$.

3.9 (**D**) $(-51.4 \times 10^{-5}\hat{j})$

A simultaneous solution yields $n_x = 4/5$ and $n_y = 3/5$. (They must both have the same sign.

3.10 a) $\cos a = \vec{V} \cdot \hat{i} / V = (1+2)/\sqrt{3^2 + 2^2} = 0.832. \qquad \therefore a = \underline{33.69^\circ}$

$$\vec{V} \cdot \hat{n} = 0. \qquad (3\hat{i} + 2\hat{j}) \cdot (n_x\hat{i} + n_y\hat{j}) = 0. \qquad \left.\begin{aligned} 3n_x + 2n_y &= 0 \\ n_x^2 + n_y^2 &= 1 \end{aligned}\right\} \quad \therefore \begin{aligned} n_y &= -\frac{3}{2}n_x \\ n_x^2 + \frac{9}{4}n_x^2 &= 1 \end{aligned}$$

$$\therefore n_x = \frac{2}{\sqrt{13}}, \quad n_y = -\frac{3}{\sqrt{13}} \quad \text{or} \quad \underline{\hat{n} = \frac{1}{\sqrt{13}}(2\hat{i} - 3\hat{j})}.$$

b) $\cos a = \vec{V}\cdot\hat{i}\,/\,V = -2/\sqrt{(-2)^2+(-8)^2} = -0.2425.$ $\quad\therefore a = \underline{104^\circ}$

$$\vec{V}\cdot\hat{n}=0. \quad (-2\hat{i}-8\hat{j})\cdot(n_x\hat{i}+n_y\hat{j})=0. \quad \left.\begin{matrix} -2n_x-8n_y=0 \\ n_x^2+n_y^2=1 \end{matrix}\right\} \therefore \begin{matrix} n_x=-4n_y \\ 16n_y^2+n_y^2=1 \end{matrix}$$

$$\therefore n_y=\frac{1}{\sqrt{17}},\ n_x=-\frac{4}{\sqrt{17}} \ \text{ or } \ \underline{\hat{n}=\frac{1}{\sqrt{17}}(-4\hat{i}+\hat{j})}.$$

c) $\cos a = \vec{V}\cdot\hat{i}\,/\,V = 5/\sqrt{5^2+(-8)^2} = 0.6202.$ $\quad\therefore a = \underline{-51.67^\circ}$

$$\vec{V}\cdot\hat{n}=0. \quad (5\hat{i}-8\hat{j})\cdot(n_x\hat{i}+n_y\hat{j})=0. \quad \left.\begin{matrix} 5n_x-8n_y=0 \\ n_x^2+n_y^2=1 \end{matrix}\right\} \therefore \begin{matrix} n_x=\frac{8}{5}n_y \\ \frac{64}{25}n_y^2+n_y^2=1 \end{matrix}$$

$$\therefore n_y=\frac{5}{\sqrt{89}},\ n_x=\frac{8}{\sqrt{89}} \ \text{ or } \ \underline{\hat{n}=\frac{1}{\sqrt{89}}(8\hat{i}+5\hat{j})}.$$

3.11 a) $\vec{V}\times d\vec{r}=0. \quad \left[(x+2)\hat{i}+xt\hat{j}\right]\times(dx\hat{i}+dy\hat{j})=0.$

$$\therefore (x+2)dy-xtdx=0 \ \text{ or } \ t\frac{xdx}{x+2}=dy.$$

Integrate: $t\int\frac{xdx}{x+2}=\int dy. \quad t\left[x-2\ell n|x+2|\right]=y+C.$

$2(1-2\ell n3)=-2+C. \quad \therefore C=0.8028.$

$\underline{t\left[x-2\ell n|x+2|\right]=y+0.8028}$

b) $\vec{V}\times d\vec{r}=0. \quad \left[xy\hat{i}-2y^2\hat{j}\right]\times(dx\hat{i}+dy\hat{j})=0.$

$$\therefore xydy+2y^2dx=0 \ \text{ or } \ \frac{2dx}{x}=-\frac{dy}{y}.$$

Integrate: $2\ell nx=-\ell n(y/C). \quad 2\ell n(1)=-\ell n(-2/C).$

$\therefore C=-2. \quad \ell nx^2=-\ell n(y/-2). \quad \therefore \underline{x^2y=-2}.$

c) $\vec{V}\times d\vec{r}=0. \quad \left[(x^2+4)\hat{i}-y^2t\hat{j}\right]\times(dx\hat{i}+dy\hat{j})=0.$

$$(x^2+4)dy+y^2tdx=0 \ \text{ or } \ \frac{tdx}{x^2+4}=-\frac{dy}{y^2}.$$

Integrate: $\frac{t}{2}\left(\tan^{-1}\frac{x}{2}+C\right)=\frac{1}{y}. \quad \frac{2}{2}\left(\tan^{-1}\frac{1}{2}+C\right)=-\frac{1}{2}.$

$\therefore C=-0.9636. \quad \underline{yt\left(\tan^{-1}\frac{x}{2}-0.9636\right)=2}$

3.12 (**C**) $\vec{a} = \frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z} = 2xy(2y\hat{i}) - y^2(2x\hat{i} - 2y\hat{j}) = -16\hat{i} - 8\hat{i} + 16\hat{j}.$

$$\therefore |a| = \sqrt{8^2 + 16^2} = 17.89 \text{ m/s}$$

3.13 a) $\frac{D\vec{V}}{Dt} = u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z} + \frac{\partial \vec{V}}{\partial t} = \underline{0}.$

b) $u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z} + \frac{\partial \vec{V}}{\partial t} = 2x(2\hat{i}) + 2y(2\hat{j}) = 4x\hat{i} + 4y\hat{j} = \underline{8\hat{i} - 4\hat{j}}$

c) $u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z} + \frac{\partial \vec{V}}{\partial t} = x^2t(2xt\hat{i} + 2yt\hat{j}) + 2xyt(2xt\hat{j} + 2zt\hat{k}) + x^2\hat{i} + 2xyj$

$$+2yzk = \underline{68\hat{i} - 100\hat{j} - 54\hat{k}}$$

d) $u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z} + \frac{\partial \vec{V}}{\partial t} = x(\hat{i} - 2yz\hat{j}) - 2xyz(-2xz\hat{j}) + tz(-2xy\hat{j} + t\hat{k}) + z\hat{k}$

$$= x\hat{i} - (2yz - 4x^2yz^2 + 2xyzt)\hat{j} + (zt^2 + z)\hat{k}$$

$$= \underline{2\hat{i} - 114\hat{j} + 15\hat{k}}$$

3.14 $\vec{\Omega} = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\hat{i} + \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\hat{j} + \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\hat{k}.$

a) $\vec{\Omega} = -\frac{1}{2}\frac{\partial u}{\partial y}\hat{k} = 20y\hat{k} = \underline{-20\hat{k}}$

b) $\vec{\Omega} = \frac{1}{2}(0-0)\hat{i} + \frac{1}{2}(0-0)\hat{j} + \frac{1}{2}(0-0)\hat{k} = \underline{0}$

c) $\vec{\Omega} = \frac{1}{2}(2zt-0)\hat{i} + \frac{1}{2}(0-0)\hat{j} + \frac{1}{2}(2yt-0)\hat{k} = \underline{6\hat{i} - 2\hat{k}}$

d) $\vec{\Omega} = \frac{1}{2}(0+2xy)\hat{i} + \frac{1}{2}(0-0)\hat{j} + \frac{1}{2}(-2yz-0)\hat{k} = \underline{-2\hat{i} + 3\hat{k}}$

3.15 The vorticity $\vec{\omega} = 2\vec{\Omega}$. Using the results of Problem 3.7:

a) $\vec{\omega} = \underline{-40\hat{i}}$ b) $\vec{\omega} = \underline{0}$ c) $\vec{\omega} = \underline{12\hat{i} - 4\hat{k}}$ d) $\vec{\omega} = \underline{-4\hat{i} + 6\hat{k}}$

3.16 a) $e_{xx} = \frac{\partial u}{\partial x} = 0, \quad e_{yy} = \frac{\partial v}{\partial y} = 0, \quad e_{zz} = \frac{\partial w}{\partial z} = 0.$

$$e_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = -20y = 20, \quad e_{xz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) = 0,$$

$$e_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = 0. \qquad \therefore \text{rate-of strain} = \begin{bmatrix} 0 & 20 & 0 \\ 20 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

b) $e_{xx} = 2,\ \ e_{yy} = 2,\ \ e_{zz} = 0.$
$e_{xy} = 0,\ \ e_{xz} = 0,\ \ e_{yz} = 0.$

$$\text{rate-of strain} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

c) $e_{xx} = 2xt = 8,\ \ e_{yy} = 2xt = 8,\ \ e_{zz} = 2yt = -4.$

$$e_{xy} = \frac{1}{2}(2yt) = -2,\ \ e_{xz} = \frac{1}{2}(0) = 0,\ \ e_{yz} = \frac{1}{2}(2zt) = 6.$$

$$\text{rate-of strain} = \begin{bmatrix} 8 & -2 & 0 \\ -2 & 8 & 6 \\ 0 & 6 & -4 \end{bmatrix}$$

d) $e_{xx} = 1,\ \ e_{yy} = -2xz = -12,\ \ e_{zz} = t = 2.$

$$e_{xy} = \frac{1}{2}(-2yz) = 3,\ \ e_{xz} = \frac{1}{2}(0) = 0,\ \ e_{yz} = \frac{1}{2}(-2xy) = 2.$$

$$\text{rate-of strain} = \begin{bmatrix} 1 & 3 & 0 \\ 3 & -12 & 2 \\ 0 & 2 & 2 \end{bmatrix}$$

3.17 a)

$$a_r = \left(10 - \frac{40}{r^2}\right)\cos q\left(\frac{80}{r^3}\right)\cos q - \left(10 + \frac{40}{r^2}\right)\frac{\sin q}{r}\left(1 - \frac{40}{r^2}\right)(-\sin q)$$
$$-\frac{1}{r}\left(10 + \frac{40}{r^2}\right)^2 \sin^2 q = (10 - 2.5)(-1)1.25(-1) = \underline{9.375\ \text{m/s}^2}.$$

$$a_q = \left(10 - \frac{40}{r^2}\right)\cos q\left(\frac{80}{r^3}\right)\sin q + \left(10 + \frac{40}{r^2}\right)\frac{\sin q}{r}\left(10 + \frac{40}{r^2}\right)\cos q$$
$$-\frac{1}{r}\left(100 - \frac{1600}{r^4}\right)\sin q\cos q = \underline{0} \qquad \text{since } \sin 180° = 0.$$

$a_f = \underline{0}.$

b) $w_r = 0,\ \ w_q = 0,\ \ w_z = \frac{1}{r}\left(-10 + \frac{40}{r^2}\right)\sin q - \frac{1}{r}\left(10 - \frac{40}{r^2}\right)(-\sin q) = 0.$

At (4, 180°) $\quad \vec{w} = \underline{0} \quad$ since $\vec{w} = 0$ everywhere.

3.18 a)

$$a_r = \left(10 - \frac{80}{r^3}\right)\cos q\left(\frac{240}{r^4}\right)\cos q - \left(10 + \frac{80}{r^3}\right)\frac{\sin q}{r}(-\sin q)\left(10 - \frac{80}{r^3}\right)$$
$$-\left(10 + \frac{80}{r^3}\right)^2 \frac{\sin^2 q}{r} = 8.75(-1)(.9375)(-1) = \underline{8.203\ \text{m/s}^2}$$

$a_q = \underline{0}$ since sin 180° = 0. $\qquad a_f = \underline{0}$ since $v_f = 0.$

b) $w_r = \underline{0},\ \ w_q = \underline{0},\ \ w_f = \underline{0},$ $\qquad$ since sin 180° = 0.

3.19 $\vec{a} = \frac{\partial \vec{V}}{\partial t} + u\cancel{\frac{\partial \vec{V}}{\partial x}} + \cancel{v}\frac{\partial \vec{V}}{\partial y} + \cancel{w}\frac{\partial \vec{V}}{\partial z} = \underline{\frac{\partial u}{\partial t}}\hat{i}.$ For steady flow $\partial u / \partial t = 0$ so that $\underline{\vec{a} = 0.}$

3.20 Assume $u(r,x)$ and $v(r,x)$ are not zero. Then, replacing z with x in the appropriate equations of Table 3.1 and recognizing that $v_\theta = 0$ and $\partial / \partial\theta = 0$:

$$a_r = v\frac{\partial v}{\partial r} + u\frac{\partial v}{\partial x} \qquad a_x = v\frac{\partial u}{\partial r} + u\frac{\partial u}{\partial x}$$

3.21 a) $u = 2(1-0)(1-e^{-t/10}) = \underline{2 \text{ m/s}}$ at $t = \infty$.

$$a_x = \frac{\partial u}{\partial t} = 2(1-0)\left(\frac{1}{10}\right)\left(e^{-t/10}\right) = \underline{0.2 \text{ m/s}^2} \text{ at } t = 0.$$

b) $u = 2(1-0.5^2)(1-e^{-t/10}) = \underline{1.875 \text{ m/s}}$ at $t = \infty$.

$$a_x = 2(1-0.5^2/2^2)\left(\frac{1}{10}e^{-t/10}\right) = \underline{0.0125 \text{ m/s}^2} \text{ at } t = 0.$$

c) $u = 2(1-2^2/2^2)(1-e^{-t/10}) = \underline{0}$ for all t.

$$a_x = 2(1-2^2/2^2)\left(\frac{1}{10}e^{-t/10}\right) = \underline{0} \text{ for all } t.$$

3.22 $\frac{DT}{Dt} = u\cancel{\frac{\partial T}{\partial x}} + \cancel{v}\frac{\partial T}{\partial y} + \cancel{w}\frac{\partial T}{\partial z} + \frac{\partial T}{\partial t} = 20(1-y^2)\left(-\frac{\pi}{100}\right)\sin\frac{\pi t}{100} = -\frac{\pi}{5}\times 0.5878$

$$= \underline{-0.3693\ ^\circ\text{C/s}}.$$

3.23 $\frac{D\rho}{Dt} = u\frac{\partial\rho}{\partial x} + v\frac{\partial\rho}{\partial y} + w\frac{\partial\rho}{\partial z} + \frac{\partial\rho}{\partial t} = 10(-1.23\times10^{-4}e^{-3000\times10^{-4}}) = \underline{-9.11\times10^{-4}\frac{\text{kg}}{\text{m}^3\cdot\text{s}}}.$

3.24 $\frac{D\rho}{Dt} = u\frac{\partial\rho}{\partial x} + v\frac{\partial\rho}{\partial y} + w\frac{\partial\rho}{\partial z} + \frac{\partial\rho}{\partial t} = 10\left(-\frac{1000}{4}\right) = \underline{-2500\frac{\text{kg}}{\text{m}^3\cdot\text{s}}}.$

3.25 $\frac{D\rho}{Dt} = u\frac{\partial\rho}{\partial x} = 4\times(.01) = \underline{0.04 \text{ kg/m}^3\cdot\text{s}}$

3.26 (**D**) $a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = u\frac{\partial u}{\partial x} = \frac{10}{(4-x)^2}\frac{\partial}{\partial x}[10(4-x)^{-2}]$

$$= \frac{10}{(4-x)^2}10(-2)(-1)(4-x)^{-3} = \frac{10}{4}\times 20\times\frac{1}{8} = 6.25 \text{ m/s}^2.$$

3.27 $\frac{D}{Dt} = \vec{V}\cdot\vec{\nabla} + \frac{\partial}{\partial t}$ observing that the dot product of two vectors $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ is $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$.

3.28 $\left.\begin{aligned} a_x &= \frac{\partial u}{\partial t} + \vec{V}\cdot\vec{\nabla}u \\ a_y &= \frac{\partial v}{\partial t} + \vec{V}\cdot\vec{\nabla}v \\ a_z &= \frac{\partial w}{\partial t} + \vec{V}\cdot\vec{\nabla}w \end{aligned}\right\} \quad \therefore \vec{a} = \frac{\partial \vec{V}}{\partial t} + (\vec{V}\cdot\vec{\nabla})\vec{V}$

3.29 Using Eq. 3.2.12:

a) $\vec{A} = \vec{a} + \frac{d^2\vec{s}}{dt^2} + 2\vec{\Omega}\times\vec{V} + \vec{\Omega}\times(\vec{\Omega}\times\vec{r}) + \frac{d\vec{\Omega}}{dt}\times\vec{r}$

$= 2(20\hat{k}\times 4\hat{i}) + 20\hat{k}\times(20\hat{k}\times 1.5\hat{i}) = \underline{160\hat{j} - 600\hat{i}\ \text{m}^2/\text{s}}$

b) $\vec{A} = 2\vec{\Omega}\times\vec{V} + \vec{\Omega}\times(\vec{\Omega}\times\vec{r}) = 2(20\hat{k}\times -20\cos 30^\circ\,\hat{j}) + 20\hat{k}\times(20\hat{k}\times 3\hat{i}) = \underline{-507\hat{i}}$

3.30 $\vec{\Omega} = \frac{2\pi}{24\times 60\times 60}\hat{k} = 7.272\times 10^{-5}\hat{k}$ rad/s.

$\vec{v} = 5(-.707\hat{i} - .707\hat{k}) = -3.535\hat{i} - 3.535\hat{k}$ m/s.

$\vec{A} = 2\vec{\Omega}\times\vec{V} + \vec{\Omega}\times(\vec{\Omega}\times\vec{r})$

$= 2\times 7.272\times 10^{-5}\hat{k}\times(-3.535\hat{i} - 3.535\hat{k}) + 7.272\times 10^{-5}\hat{k}\times$

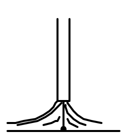
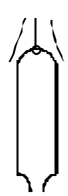
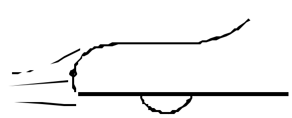
$[7.272\times 10^{-5}\hat{k}\times 6\times 10^6(-.707\hat{i} + .707\hat{k})] = \underline{-51.4\times 10^{-5}\hat{j} + 0.0224\hat{i}\ \text{m/s}^2}.$

Note: We have neglected the acceleration of the earth relative to the sun since it is quite small (it is $d^2\vec{s}/dt^2$). The component $(-51.4\times 10^{-5}\hat{j})$ is the Coriolis acceleration and causes air motions to move c.w. or c.c.w. in the two hemispheres.

3.31 a) two-dimensional *(r, z)* b) two-dimensional *(x, y)*
c) two-dimensional *(r, z)* d) two-dimensional *(r, z)*
e) three-dimensional *(x, y, z)* f) three-dimensional *(x, y, z)*
g) two-dimensional *(r, z)* h) one-dimensional *(r)*

3.32 Steady: a, c, e, f, h Unsteady: b, d, g

3.33 b. It is an unsteady plane flow.

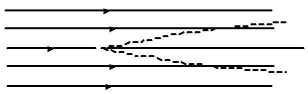
3.34 a)  d)  e) 

3.35 f, h

3.36 a) inviscid. b) inviscid. c) inviscid.
d) viscous inside the boundary layer.
e) viscous inside the boundary layers and separated regions.
f) viscous. g) viscous. h) viscous.

3.37 d and e. Each flow possesses a stagnation point.

3.38



3.39 (**C**) The only velocity component is $u(x)$. We have neglected $v(x)$ since it is quite small. If $v(x)$ in not negligible, the flow would be two-dimensional.

3.40 $\text{Re} = VL/n = 2 \times .015 / .77 \times 10^{-6} = 39\,000.$ $\therefore$ Turbulent.

3.41 $\text{Re} = \dfrac{VL}{n} = .2 \times .8 / 1.4 \times 10^{-5} = 11\,400.$ $\therefore$ Turbulent.

3.42 $\text{Re} = \dfrac{VL}{n} = \dfrac{4 \times .06}{1.7 \times 10^{-5}} = 14\,100.$ $\therefore$ Turbulent.
Note: We used the smallest dimension to be safe!

3.43 a) $\text{Re} = \dfrac{VD}{n} = \dfrac{1.2 \times 0.01}{1.51 \times 10^{-5}} = 795.$ Always laminar.

b) $\text{Re} = \dfrac{VD}{n} = \dfrac{1.2 \times 1}{1.51 \times 10^{-5}} = 79\,500.$ May not be laminar.

3.44 $\text{Re} = 3 \times 10^5 = \dfrac{Vx_T}{n}.$ $n = m/r$ where $m = m(T)$.

a) $T = 223$ K or $-50°$C. $\therefore m = 1.5 \times 10^{-5}\ \text{N} \cdot \text{s/m}^2.$

$$\therefore n = \frac{1.5 \times 10^{-5}}{.3376 \times 1.23} = 2.5 \times 10^{-5}\ \text{m}^2/\text{s}.$$

$$3 \times 10^5 = \frac{900 \times 1000 x_T}{3600 \times 2.5 \times 10^{-5}}. \qquad \therefore x_T = 0.03 \text{ m} \quad \text{or} \quad \underline{3 \text{ cm}}$$

b) $T = -48°$F. $\therefore m = 3.3 \times 10^{-7}$ lb-sec/ft². $n = \dfrac{3.3 \times 10^{-7}}{.00089} = 3.7 \times 10^{-4}$ ft²/sec.

$$3 \times 10^5 = \frac{600 \times 5280 x_T}{3600 \times 3.7 \times 10^{-4}}. \qquad \therefore x_T = 0.13' \quad \text{or} \quad \underline{1.5''}$$

3.45 Assume the flow is parallel to the leaf. Then $3 \times 10^5 = Vx_T \,/\, n$.

$\therefore x_T = 3 \times 10^5 n \,/\, V = 3.5 \times 10^5 \times 1.4 \times 10^{-4} \,/\, 6 = 8.17$ m.

The flow is expected to be <u>laminar</u>.

3.46 a) $M = \dfrac{V}{c} = \dfrac{100}{\sqrt{1.4 \times 287 \times 236}} = 0.325.$ For accurate calculations the flow is <u>compressible</u>. Assume incompressible flow if an error of 4%, or so, is acceptable.

b) $M = \dfrac{V}{c} = \dfrac{80}{\sqrt{1.4 \times 287 \times 288}} = 0.235.$ $\therefore$ Assume <u>incompressible</u>.

c) $M = \dfrac{V}{c} = \dfrac{100}{\sqrt{1.4 \times 287 \times 373}} = 0.258.$ $\therefore$ Assume <u>incompressible</u>.

3.47 $\dfrac{Dr}{Dt} = u\dfrac{\P r}{\P x} + v\dfrac{\P r}{\P y} + w\dfrac{\P r}{\P z} + \dfrac{\P r}{\P t} = 0.$ For a steady, plane flow

$\P r \,/\, \P t = 0$ and $w = 0$. Then

$$u\frac{\P r}{\P x} + v\frac{\P r}{\P y} = 0.$$

3.48 $\dfrac{Dr}{Dt} = u\cancel{\dfrac{\P r}{\P x}} + v\cancel{\dfrac{\P r}{\P y}} + \cancel{w}\dfrac{\P r}{\P z} + \cancel{\dfrac{\P r}{\P t}} = 0.$ $\therefore$ incompressible.

3.49 (**B**) $\dfrac{V^2}{2} = \dfrac{p}{r} = \dfrac{g_{water} h}{r_{air}} = \dfrac{9810 \times 0.800}{1.23}.$ $\therefore V = 113$ m/s.

3.50 $\dfrac{V^2}{2} = \dfrac{p}{r}.$ Use $r = 0.0021$ slug/ft^3.

a) $v = \sqrt{2p \,/\, r} = \sqrt{2 \times .3 \times 144 / .0021} =$ <u>203 ft/sec</u>.

b) $v = \sqrt{2p \,/\, r} = \sqrt{2 \times .9 \times 144 / .0021} =$ <u>351 ft/sec</u>.

c) $v = \sqrt{2p \,/\, r} = \sqrt{2 \times .09 \times 144 / .0021} =$ <u>111 ft/sec</u>.

3.51 $p = r\dfrac{V^2}{2} = 1.23\left(\dfrac{120 \times 1000}{3600}\right)^2 / \; 2 = 683$ Pa.

$\therefore F = pA = 683\, p \times 0.075^2 =$ <u>12.1 N</u>.

3.52 $\dfrac{V^2}{2} + \dfrac{p}{r} = 0.$ $\therefore V = \sqrt{\dfrac{-2p}{r}} = \sqrt{\dfrac{2 \times 2000}{1.23}} =$ <u>57.0 m/s</u>

3.53 (**C**) $\frac{V_1^2}{2g}+\frac{p}{g}=\frac{V_2^2}{2g}$. $\frac{V_1^2}{2g}+0.200=0.600$. $\therefore V=\sqrt{2\times 9.81\times 0.400}=2.80$ m/s.

3.54 (**B**) The manometer reading h implies:

$\frac{V_1^2}{2}+\frac{p_1}{r}=\frac{V_2^2}{2}+\frac{p_2}{r}$ or $V_2^2=\frac{2}{1.13}(60-10.2)$. $\therefore V_2=\underline{9.39\text{ m/s}}$ The temperature (the viscosity of the water) and the diameter of the pipe are not needed.

3.55 a) $\frac{V^2}{2}+\frac{p}{r}=\cancel{\frac{V_0^2}{2}}+\frac{p_o}{r}$. $\frac{(-10x)^2}{2}+\frac{p}{r}=\frac{p_o}{r}$. $\therefore p=\underline{p_o-50x^2 r}$

b) $\frac{V^2}{2}+\frac{p}{r}=\cancel{\frac{V_0^2}{2}}+\frac{p_o}{r}$. $\frac{(10y)^2}{2}+\frac{p}{r}=\frac{p_o}{r}$. $\therefore p=\underline{p_o-50y^2 r}$

3.56 $\frac{V^2}{2}+\frac{p}{r}=\frac{U_\infty^2}{2}+\cancel{\frac{p_\infty}{r}}$.

a) $v_q=0$ and $q=180^\circ$: $v_r=U_\infty(1-r_c^2/r^2)(-1)$.

$$\therefore p=\frac{r}{2}\left(U_\infty^2-v_r^2\right)=\underline{\frac{r}{2}U_\infty^2\left[2\frac{r_c^2}{r^2}-\left(\frac{r_c}{r}\right)^4\right]}.$$

b) Let $r=r_c$: $\underline{p_T=\frac{r}{2}U_\infty^2}$

c) $v_r=0$ and $r=r_c$: $v_q=-U_\infty 2\sin q$. $\therefore p=\frac{r}{2}\left(U_\infty^2-v_q^2\right)=\underline{\frac{r}{2}U_\infty^2\left[1-4\sin^2 q\right]}$

d) Let $q=90^\circ$: $\underline{p_{90}=-\frac{3}{2}rU_\infty^2}$

3.57 $\frac{V^2}{2}+\frac{p}{r}=\frac{U_\infty^2}{2}+\cancel{\frac{p_\infty}{r}}$.

a) $v_q=0$ and $q=180^\circ$: $p=\frac{r}{2}\left(U_\infty^2-v_r^2\right)=\underline{\frac{r}{2}U_\infty^2\left[2\left(\frac{r_c}{r}\right)^3-\left(\frac{r_c}{r}\right)^6\right]}$.

b) Let $r=r_c$: $\underline{p_T=\frac{1}{2}rU_\infty^2}$.

c) $v_r=0$ and $r=r_c$: $p=\frac{r}{2}\left(U_\infty^2-v_q^2\right)=\underline{\frac{r}{2}U_\infty^2\left[1-4\sin^2 q\right]}$

d) Let $q=90^\circ$: $\underline{p_{90}=-\frac{3}{2}rU_\infty^2}$

3.58 $\dfrac{V^2}{2}+\dfrac{p}{r}=\dfrac{U_\infty^2}{2}+\cancel{\dfrac{p_\infty}{r}}.$

a) $p=\dfrac{r}{2}\left(U_\infty^2-u^2\right)=\dfrac{r}{2}\left[10^2-\left(10+\dfrac{20p}{2px}\right)^2\right]=50\,r\left[1-\left(1+\dfrac{1}{x}\right)^2\right]$

$$=-50\,r\left(\frac{2}{x}+\frac{1}{x^2}\right)$$

b) $u=0$ when $x=-1$. $\quad p_{-1}=-50\,r(-2+1)=\underline{50\,r}$

c) $p=\dfrac{r}{2}\left(U_\infty^2-u^2\right)=\dfrac{r}{2}\left[30^2-\left(30+\dfrac{60p}{2p\,x}\right)^2\right]=450\,r\left[1-\left(1+\dfrac{1}{x}\right)^2\right]=-450\,r\left(\dfrac{2}{x}+\dfrac{1}{x^2}\right)$

d) $u=0$ when $x=-1$. $\quad p_{-1}=-450\,r(-2+1)=\underline{450\,r}$

3.59 $\dfrac{V_1^2}{2}+\dfrac{p_1}{r}=\dfrac{V_2^2}{2}+\dfrac{p_2}{r}.\quad V_1=0$ and $p_1-p_2=20$ kPa.

$V_2^2=\dfrac{2}{r}\left(p_1-p_2\right)=\dfrac{2}{1000}(20\,000)=40.\quad \therefore V_2=\underline{6.32\text{ m/s}}$

3.60 Assume the velocity in the plenum is zero. Then

$\dfrac{V_1^2}{2}+\dfrac{p_1}{r}=\dfrac{V_2^2}{2}+\dfrac{p_2}{r}$ or $V_2^2=\dfrac{2}{1.13}(60-10.2).\quad \therefore V_2=\underline{9.39\text{ m/s}}$

We found $r=1.13\text{ kg/m}^3$ in Table B.2.

3.61 Bernoulli from the stream to the pitot probe: $p_T=r\dfrac{V^2}{2}+p.$

Manometer: $p_T+g\,H-g_{Hg}H-gh=p-g\,h.$

Then, $r\dfrac{V^2}{2}+p+g\,H-g_{Hg}H=p.\quad \therefore V^2=\dfrac{g_{Hg}-g}{r}(2H)$

a) $V^2=\dfrac{(13.6-1)9800}{1000}(2\times0.04).\quad \therefore V=\underline{3.14\text{ m/s}}$

b) $V^2=\dfrac{(13.6-1)9800}{1000}(2\times0.1).\quad \therefore V=\underline{4.97\text{ m/s}}$

c) $V^2=\dfrac{(13.6-1)62.4}{1.94}(2\times2/12).\quad \therefore V=\underline{11.62\text{ fps}}$

d) $V^2=\dfrac{(13.6-1)62.4}{1.94}(2\times4/12).\quad \therefore V=\underline{16.44\text{ fps}}$

3.62 The pressure at 90° from Problem 3.56 is $p_{90} = -3rU_\infty^2/2$. The pressure at the stagnation point is $p_T = rU_\infty^2/2$. The manometer provides: $p_T - gH = p_{90}$

$$\frac{1}{2}\times 1.204U_\infty^2 - 9800\times 0.04 = -\frac{3}{2}\times 1.204U_\infty^2. \qquad \therefore U_\infty = \underline{12.76 \text{ m/s}}$$

3.63 The pressure at 90° from Problem 3.57 is $p_{90} = -3rU_\infty^2/2$. The pressure at the stagnation point is $p_T = rU_\infty^2/2$. The manometer provides: $p_T - gH = p_{90}$

$$\frac{1}{2}\times 1.204U_\infty^2 - 9800\times 0.04 = -\frac{3}{2}\times 1.204U_\infty^2. \qquad \therefore U_\infty = \underline{12.76 \text{ m/s}}$$

3.64 Assume an incompressible flow with point 1 outside in the room where $p_1 = 0$ and $v_1 = 0$. The Bernoulli's equation gives, with $p_2 = g_w h_2$,

$$\cancel{\frac{V_1^2}{2}} + \cancel{\frac{p_1}{r}} = \frac{V_2^2}{2} + \frac{p_2}{r}.$$

$$\text{a) } 0 = \frac{V_2^2}{2} + \frac{-9800\times 0.02}{1.204}. \qquad \therefore V_2 = \underline{18.04 \text{ m/ s}}$$

$$\text{b) } 0 = \frac{V_2^2}{2} + \frac{-9800\times 0.08}{1.204}. \qquad \therefore V_2 = \underline{36.1 \text{ m/ s}}$$

$$\text{c) } 0 = \frac{V_2^2}{2} + \frac{-62.4\times 1/\ 12}{0.00233}. \qquad \therefore V_2 = \underline{66.8 \text{ fps}}$$

$$\text{d) } 0 = \frac{V_2^2}{2} + \frac{-62.4\times 4/\ 12}{0.00233}. \qquad \therefore V_2 = \underline{133.6 \text{ fps}}$$

3.65 Assume incompressible flow ($V < 100$ m/s) with point 1 outside the wind tunnel where $p_1 = 0$ and $V_1 = 0$. Bernoulli's equation gives

$$0 = \frac{V_2^2}{2} + \frac{p_2}{r}. \qquad \therefore p_2 = -\frac{1}{2}rV_2^2$$

$$\text{a) } r = \frac{p}{RT} = \frac{90}{0.287\times 253} = 1.239 \text{ kg/ m}^3. \quad \therefore p_2 = -\frac{1}{2}\times 1.239\times 100^2 = \underline{-6195 \text{ Pa}}$$

$$\text{b) } r = \frac{p}{RT} = \frac{95}{0.287\times 273} = 1.212 \text{ kg/ m}^3. \quad \therefore p_2 = -\frac{1}{2}\times 1.212\times 100^2 = \underline{-6060 \text{ Pa}}$$

$$\text{c) } r = \frac{p}{RT} = \frac{92}{0.287\times 293} = 1.094 \text{ kg/ m}^3. \quad \therefore p_2 = -\frac{1}{2}\times 1.094\times 100^2 = \underline{-5470 \text{ Pa}}$$

$$\text{d) } r = \frac{p}{RT} = \frac{100}{0.287\times 313} = 1.113 \text{ kg/ m}^3. \quad \therefore p_2 = -\frac{1}{2}\times 1.113\times 100^2 = \underline{-5566 \text{ Pa}}$$

3.66 (**A**) $\cancel{\frac{V_1^2}{2g}} + \frac{p_1}{g} = \frac{V_2^2}{2g} + \cancel{\frac{p_2}{g}}. \qquad \frac{800000}{9810} = \frac{V_2^2}{2\times 9.81}. \qquad \therefore V_2 = 40 \text{ m/s}.$

3.67 a) $p_A = \gamma h = 9800 \times 4 = 39\ 200$ Pa, $V_A = 0$. Using $h_A = h_2$,

$$\frac{V_A^2}{2g} + \frac{p_A}{\gamma} + h_A = \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + h_2. \quad p_2 = p_A - \frac{V_2^2}{2g}\gamma$$

$$= 39\ 200 - \frac{14^2}{2 \times 9.81} \times 9800 = \underline{-58\ 700\ \text{Pa}}$$

b) $p_B = 0$ and $V_B = 0$. Bernoulli's eq. gives, with the datum through the pipe,

$$\frac{V_B^2}{2g} + \frac{p_B}{\gamma} + h_B = \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + h_2. \quad p_2 = \left(4 - \frac{14^2}{2 \times 9.81}\right)9800 = \underline{-58\ 700\ \text{Pa}}$$

3.68 Bernoulli: $\frac{V_2^2}{2g} + \frac{p_2}{\gamma} = \frac{V_1^2}{2g} + \frac{p_1}{\gamma}$

Manometer: $p_1 + \gamma z + \gamma_{Hg} H - \gamma H - \gamma z = \frac{V_2^2}{2g}\gamma + p_2$

Substitute Bernoulli's into the manometer equation:

$$p_1 + (\gamma_{Hg} - \gamma)H = \frac{V_1^2}{2g}\gamma + p_1.$$

a) Use H = 0.01 m: $\frac{V_1^2 \times 9800}{2 \times 9.81} = (13.6 - 1)9800 \times 0.01 \quad \therefore V_1 = \underline{1.572\ \text{m/s}}$

Substitute into Bernoulli:

$$p_1 = \frac{V_2^2 - V_1^2}{2g}\gamma = \frac{20^2 - 1.572^2}{2 \times 9.81} \times 9800 = \underline{198\ 600\ \text{Pa}}$$

b) Use H = 0.05 m: $\frac{V_1^2 \times 9800}{2 \times 9.81} = (13.6 - 1)9800 \times 0.05 \quad \therefore V_1 = \underline{3.516\ \text{m/s}}$

Substitute into Bernoulli:

$$p_1 = \frac{V_2^2 - V_1^2}{2g}\gamma = \frac{20^2 - 3.516^2}{2 \times 9.81} \times 9800 = \underline{193\ 600\ \text{Pa}}$$

c) Use H = 0.1 m: $\frac{V_1^2 \times 9800}{2 \times 9.81} = (13.6 - 1)9800 \times 0.1 \quad \therefore V_1 = \underline{4.972\ \text{m/s}}$

Substitute into Bernoulli:

$$p_1 = \frac{V_2^2 - V_1^2}{2g}\gamma = \frac{20^2 - 4.972^2}{2 \times 9.81} \times 9800 = \underline{187\ 400\ \text{Pa}}$$

3.69 Bernoulli across nozzle: $\frac{V_1^2}{2} + \frac{p_1}{r} = \frac{V_2^2}{2} + \frac{p_2}{r}$. $\therefore V_2 = \sqrt{2p_1/r}$

Bernoulli to max. height: $\frac{V_1^2}{2g} + \frac{p_1}{g} + h_1 = \frac{V_2^2}{2g} + \frac{p_2}{g} + h_2$. $\therefore h_2 = p_1/g$.

a) $V_2 = \sqrt{2p_1/r} = \sqrt{2 \times 700\,000/1000} = \underline{37.42 \text{ m/s}}$
$h_2 = p_1/g = 700\,000/9800 = \underline{71.4 \text{ m}}$

b) $V_2 = \sqrt{2p_1/r} = \sqrt{2 \times 1\,400\,000/1000} = \underline{52.92 \text{ m/s}}$
$h_2 = p_1/g = 1\,400\,000/9800 = \underline{142.9 \text{ m}}$

c) $V_2 = \sqrt{2p_1/r} = \sqrt{2 \times 100 \times 144/1.94} = \underline{121.8 \text{ fps}}$
$h_2 = p_1/g = 100 \times 144/62.4 = \underline{231 \text{ ft}}$

d) $V_2 = \sqrt{2p_1/r} = \sqrt{2 \times 200 \times 144/1.94} = \underline{172.3 \text{ fps}}$
$h_2 = p_1/g = 200 \times 144/62.4 = \underline{462 \text{ ft}}$

3.70 a) Apply Bernoulli's eq. from the surface to a point on top of the downstream flow:

$$\frac{V_1^2}{2g} + \frac{p_1}{g} + h_1 = \frac{V_2^2}{2g} + \frac{p_2}{g} + h_2. \quad \therefore \underline{V_2 = \sqrt{2g(H-h)}}$$

b) Apply Bernoulli's eq. from a point near the bottom upstream to a point on the bottom of the downstream flow:

$$\frac{V_1^2}{2g} + \frac{p_1}{g} + h_1 = \frac{V_2^2}{2g} + \frac{p_2}{g} + h_2.$$

Using $p_1 = gH$, $p_2 = gh$ and $h_1 = h_2$, $\underline{V_2 = \sqrt{2g(H-h)}}$

3.71 $\frac{V_1^2}{2} + \frac{p_1}{r} = \frac{V_2^2}{2} + \frac{p_2}{r}$. $p_2 = -100\,000$ Pa, the lowest possible pressure.

a) $\frac{600\,000}{1000} = \frac{V_2^2}{2} - \frac{100\,000}{1000}$. $\therefore V_2 = \underline{37.4 \text{ m/s}}$.

b) $\frac{300\,000}{1000} = \frac{V_2^2}{2} - \frac{100\,000}{1000}$. $\therefore V_2 = \underline{28.3 \text{ m/s}}$.

c) $\frac{80\times 144}{1.94}=\frac{V_2^2}{2}-\frac{14.7\times 144}{1.94}.\qquad \therefore V_2 = \underline{118.6\text{ ft/sec}}.$

d) $\frac{40\times 144}{1.94}=\frac{V_2^2}{2}-\frac{14.7\times 144}{1.94}.\qquad \therefore V_2 = \underline{90.1\text{ ft/sec}}.$

3.72 A water system must never have a negative pressure, since a leak could ingest impurities. $\therefore$ The least pressure is zero gage.

$$\frac{V_1^2}{2}+\frac{p_1}{\rho}+gz_1=\frac{V_2^2}{2}+\frac{p_2}{\rho}+gz_2.\qquad V_1=V_2.\qquad \text{Let } z_1=0, \text{ and } p_2=0.$$

$$\frac{500\,000}{1000}=9.81\,z_2.\qquad \therefore z_2=\underline{51.0\text{ m}}.$$

3.73

a) $p_1=\frac{\rho}{2}\left(V_2^2-V_1^2\right)=\frac{1000}{2}\left(2^2-10^2\right)=\underline{-48\,000\text{ Pa}}$

b) $p_1=\frac{\rho}{2}\left(V_2^2-V_1^2\right)=\frac{902}{2}\left(2^2-10^2\right)=\underline{-43300\text{ Pa}}$

c) $p_1=\frac{\rho}{2}\left(V_2^2-V_1^2\right)=\frac{680}{2}\left(2^2-10^2\right)=\underline{-32600\text{ Pa}}$

d) $p_1=\frac{\rho}{2}\left(V_2^2-V_1^2\right)=\frac{1.23}{2}\left(2^2-10^2\right)=\underline{-59.0\text{ Pa}}$

3.74 $\frac{V_1^2}{2}+\frac{p_1}{\rho}=\frac{V_2^2}{2}+\frac{p_2}{\rho}.\qquad p_1=\frac{\rho}{2}\left(V_2^2-V_1^2\right)=\frac{1.23}{2}\left(2^2-8^2\right)=\underline{-36.9\text{ Pa}}$

3.75 (**D**) $p_1=\frac{\rho}{2}\left(V_2^2-V_1^2\right)=\frac{902}{2}\left(30^2-15^2\right)=\underline{304400\text{ Pa}}$

3.76 Apply Bernoulli's equation between the exit (point 2) where the radius is R and a point 1 in between the exit and the center of the tube at a radius r less than R:

$$\frac{V_1^2}{2}+\frac{p_1}{\rho}=\frac{V_2^2}{2}+\frac{p_2}{\rho}.\qquad \therefore p_1=\rho\,\frac{V_2^2-V_1^2}{2}.$$

Since $V_2 < V_1$, we see that p_1 is negative (a vacuum) so that the envelope would tend to rise due to the negative pressure over most of its area (except for a small area near the end of the tube).

3.77 $\text{Re}=\frac{VD}{\nu}$. For air $\nu \cong 1.5\times 10^{-5}$. Use reasonable dimensions from your experience!

a) $\text{Re} = \dfrac{20 \times 0.03}{1.5 \times 10^{-5}} = 4 \times 10^4.$ $\therefore$ Separate

b) $\text{Re} = \dfrac{20 \times 0.005}{1.5 \times 10^{-5}} = 6700.$ $\therefore$ Separate

c) $\text{Re} = \dfrac{20 \times 2}{1.5 \times 10^{-5}} = 2.7 \times 10^6.$ $\therefore$ Separate

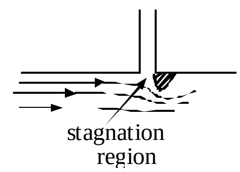
d) $\text{Re} = \dfrac{5 \times 0.002}{1.5 \times 10^{-5}} = 670.$ $\therefore$ Separate

e) $\text{Re} = \dfrac{20 \times 2}{1.5 \times 10^{-5}} = 2.7 \times 10^6.$ $\therefore$ Separate

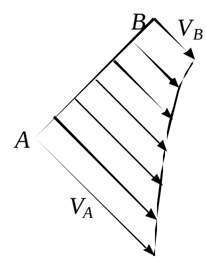
f) $\text{Re} = \dfrac{100 \times 3}{1.5 \times 10^{-5}} = 2 \times 10^7.$

$\therefore$ It will tend to separate, except streamlining the components eliminates separation.

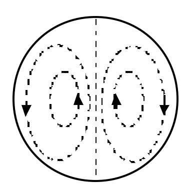
3.78 A burr downstream of the opening will create a region that acts similar to a stagnation region thereby creating a high pressure since the velocity will be relatively low in that region.



3.79 $\Delta p = r\dfrac{V^2}{R}\Delta n = 1000\dfrac{10^2}{0.05} \times 0.02 = \underline{40\ 000 \text{ Pa}}$ Along AB, we expect $V_A > 10$ m / s and $V_B < 10$ m / s.



3.80 The higher pressure at B will force the fluid toward the lower pressure at A, especially in the wall region of slow moving fluid, thereby causing a secondary flow normal to the pipe's axis. This results in a relatively high loss for an elbow.



3.81 Refer to Bernoulli's equation: $\dfrac{V_1^2}{2} + \dfrac{p_1}{r} = \dfrac{V_2^2}{2} + \dfrac{p_2}{r}$

$p_A > p_B$ since $V_A < V_B$

$p_C < p_D$ since $V_C > V_D$

$p_B > p_D$ since $V_D > V_B$